Wealth Distribution Evolution in an Agent-based Computational Economics

Victor Romanov, Dmitry Yakovlev and Anna Lelchuk

Abstract In this paper we study the modification of wealth distribution among the customers during quite a long period of time in the model - several model years. During this time customers get their income in forms of salary depending on enterprise production volume and assortment, or redundancy payments. As a part of the study it was detected that whilst the initial wealth distribution was uniform a strong non-uniformity arises after several years in the model.

The model includes the following interacting agent classes: customer, bank, labor market, state, enterprise, market, university, and mass media. The model also allows us to evaluate the relations among the efficiency of enterprises' investment strategies, tax level and customer's prosperity and unemployment level. The possibility of obtaining a new specialty by a fired agent for the purpose of stabilization and increasing his profit and improve standard of life is considered in the paper as well.

1 Introduction

The problems of multi-agent modeling in economics attract more and more scientists' attention, since the pioneer article by Leigh Tesfatsion [18]. Multi-agent simulation for food market is considered in the paper [5]. The related topics on negotiations are discussed in the article [13]. In this paper we are particularly inter-

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ested in enterprise simulation and consumer behavior issues partly covered in the articles [11,8].

The problem of inequality of wealth distribution in society has been considered in scientific literature for quite a long time. Wealth distribution in artificial and real economics has attracted great attention in the research community since the last decades of the XIXth century, starting with Vilfredo Pareto [12]. A brilliant analytical review on this topic is contained in the article [10].

The goal of our paper is to study how the customers' wealth distribution function is modified during their lifetime. The wealth is formed by incoming salary or redundancy payments and depends on the variability of demand and on the corresponding modified enterprise production volume and assortment. It has to be noted that "wealth" is here understood differently than its common meaning. By customers' wealth we shall basically mean the cumulative sum of customers' money composed of the initial wealth endowed in a random manner according to the predefined law, and the amount of the following regular earnings (salary or redundancy relieves) minus cost of the purchases made.

To make sure that the results are robust, in this research we adduce the results of wealth distribution simulation in the virtual economics in cases from 300 to 1000 customers, and 5 and 10 enterprises respectively. We discovered the change of distribution function according to the scheme: rectangular - normal - gamma - normal - rectangular during a twelve-year period. We also found out that a group of customers with a much higher income (5-6 times higher than an average income), and a gap dividing the two groups of customers appear once we set some critical tax level (24%).

2 Equilibrium Wealth Distribution Models

One of the earliest works devoted to the problem of inequality based on multi-agent approach was the one of Angle [3]. The evolution of wealth distribution between two agents - winner and looser - takes place in the model of Angle.

The model demonstrated that the process leads to a gamma distribution.

Different kinds of stable probability distribution of agents concentration were discovered in the ants model by Kirman (1993) and described by S. Alfarano, *et al.* (2005), (2008) [1-2]. In the model the ant colony is inhabited by N agents, each in one of two possible states. The average number of neighbours per agent is D, relative communication rate is D/N. Transition rate from one state to another depends on the concentration n/N and $(N-n)/N$ of agents in the appropriate state with strength random state transition strength coefficient a, and on presence of interaction among agents $n(N-n)$ with strength coefficient b. Markov chain approach based on the evolution equation for transition $w(x,t/x_0,t_0)$ probability density was applied.

The state of system depends on the concentration of agents in either of the two states. Equilibrium state refers to stationary distribution of the process according to the time the system spends in the state x.

According to the general solution well known from handbook [14] we get

$$w_{st}(x) = \frac{\Gamma(2\varepsilon)}{2^{2\varepsilon-1}\Gamma(\varepsilon)^2}(1-x^2)^{\varepsilon-1}, \varepsilon = \frac{a}{b}. \quad (1)$$

The remarkable characteristics of this expression is that for $\varepsilon > 1$ the function is unimodal, for $\varepsilon < 1$ it is bimodal, for $\varepsilon = 1$ it is uniform, and for $\varepsilon >> 1$ the equilibrium distribution converges to Gaussian.

The approach from the point of view of classical statistical mechanics and thermodynamics was applied by Victor Yakovenko *et al.* [7,4], who offered the model that included agents (particles) exchanging a constant amount of energy (money). They studied probability distribution of money and income for ensemble of economic agents. The result of income distribution interpolate between exponential Boltzmann-Gibbs law for average and low income.

In the trading model [6] propensity to save is assigned to all agents; they save λ-fraction $(0 \leq \lambda < 1)$ of wealth during the trade. Wealth distribution in this model converges to Gamma distribution.

Silver J. *et al.* [16] describe a market consisting of many agents with Cobb-Douglas preferences in the case of two goods. They describe mathematically that Gamma distribution arises for a broader class of preference distributions in the limit of large numbers of individual agents.

A brief review of researches on the exchange process modeling in multi-agent systems demonstrates that in the different models arise the same set of available probability distribution functions which describe interacting agents in multi-agent systems: exponential, Gaussian, gamma, Pareto. With different values of model parameters, this probability distributions may turn from one form into another. The results obtained in our work do not contradict the research made by other authors. The specific feature of our model is that such transition happens during one cycle of model functioning.

The wealth distribution evolution in our model may be explained by changing of drift and diffusion coefficients in time.

The plot of the finite difference approximation time dependent drift $\frac{d<x>}{dt}$ and diffusion $\frac{d<x^2>}{dt}$ coefficients are presented in Figures 1 and 2, respectively.

We suppose that in our model transition from one to another kind of probability distribution function in the cycle of model operation happens due to non-linear type of dependences and positive feedback.

To illustrate this proposition let us consider the following non-linear Fokker-Planck equation [19]:

$$\frac{\partial w(x,t)}{\partial t} = \frac{\partial(\alpha x + \beta\langle x(t)\rangle w(x,t))}{\partial x} + D\frac{\partial^2 w(x,t)}{\partial x^2},$$

where $\langle x(t)\rangle = \int_{-\infty}^{\infty} x w(x,t)dx$, and α, β, and D are constant.

Assuming initial δ-function wealth distribution $w(x,0) = \delta(x - x_0)$ stable probability distribution "moves" according to $\langle x(t)\rangle$ changing in time and gets modified

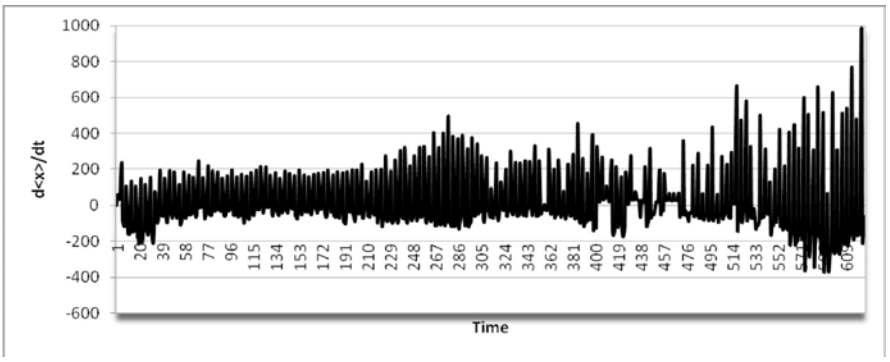

Fig. 1 Drift coefficient.

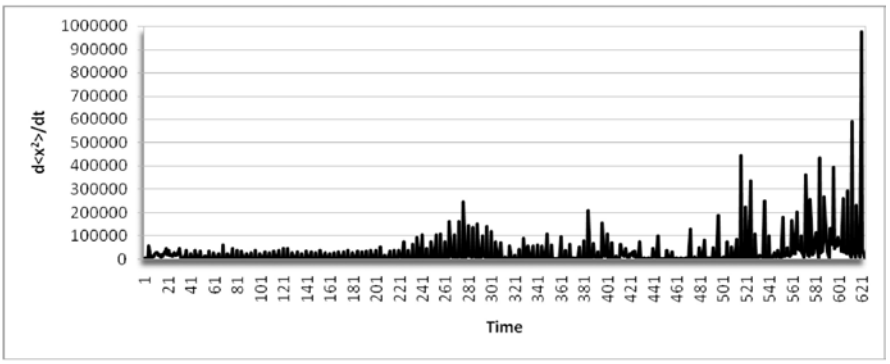

Fig. 2 Diffusion coefficient.

due to dependence of σ on t. So in case of non-linear Fokker-Planck equations we may expect evolution of probability distribution in time.

3 Model Architecture

The key issues in designing adequate and effective model of economy are to understand and analyze the following interrelated phenomena:

- wealth possessed by customers and its repartition during their lifetime;
- effect of demand on production level;
- dependence of model's macroeconomic indices on tax level and the enterprise strategy;
- impact of the state's tax policy on macroeconomic indices;
- relationship between enterprise investment strategy and such macroeconomic indicators as tax level, employed population as well as university fee;

- dependence of the amount of working population on the enterprise's investment strategy, tax level and university fee.

While doing this research the authors presumed that the changing composite demand of the economic agents is the major driving force of the economic development. We also believe that economic efficiency is mainly determined by the correlation of the current output with the varying consumer needs and demand.

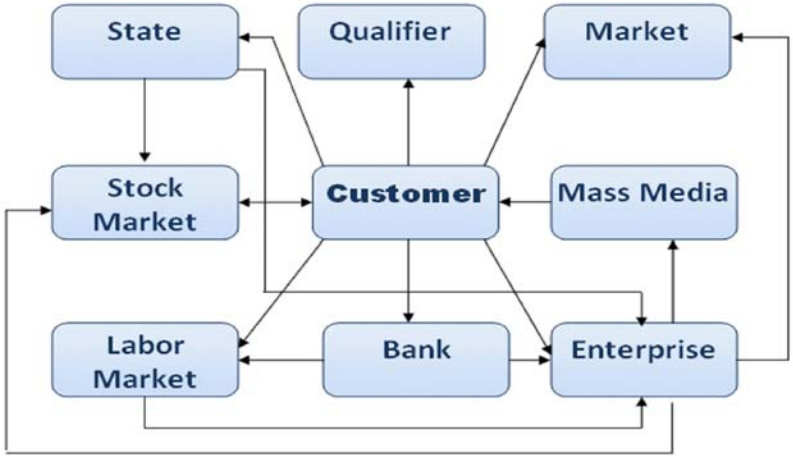

Fig. 3 Classes diagram.

Classes diagram of the model actors is presented in Figure 3. Initially the diagram was created in Altova UModel but here we present a simplified scheme of model's classes.

- ENTERPRISE acts as a seller on the market, a client of the bank, a taxpayer and an employer;
- MARKET provides customers with an access to goods produced by enterprises to customers and in such a way keeps the dynamic balance between the aggregate demand and aggregate supply, and forms an equilibrium price on the base of bargaining;
- STATE collects taxes from the enterprises and pays redundancy relieves;
- BANK (FIRST_NAT_BANK) organizes money transactions among agents;
- LABOR_MARKET lists vacancies that enterprises possess and send messages about them to the job seekers, and provides equilibrium between labor demand and supply;
- QUALIFIER is an educational institution that performs an employee's training and requalification for a definite fee;

- MASS MEDIA receives orders from the enterprises for an advertising campaign, then selects a random sample of customers with aim to increase the value of the subjective utility function of the advertised product;
- Enterprises make initial public stock offerings on the STOCK MARKET and then provides the buy/sell operations among the issuers and the shareholders. The model interface is presented in the Figure 4.

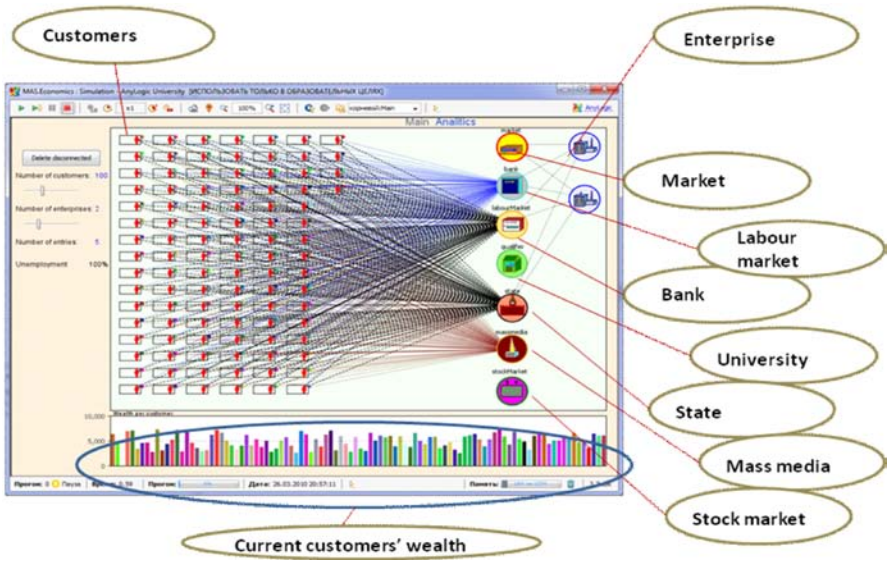

Fig. 4 The model interface.

The process of model's dynamics is determined by three main types of agents' activities:

- Agent job placement;
- Customer behavior;
- Enterprise investment strategy;
- Current wealth distribution among customers.

4 Agent Job Placement

A customer may be in one of four possible states: employed, unemployed, student, dead. In the initial moment all the customers are unemployed and get redundancy payments from the state. All of them send messages to the labor market and are registered there as unemployed. At the same time the enterprises send messages to the labor market with the vacancies they possess. Next, the agents get employed

on one of the enterprises and stop receiving the redundancy relief if the number of vacancies is equal to the number of unemployed; otherwise a customer may remain unemployed. As employees they obtain salary according to their degree and qualification.

In case of reduced production level an enterprise can fire an employee so he registers at the labor market again. If he cannot find a vacancy according to qualification he sends another message to the labor market about his willingness to get a new qualification for a certain fee. As he gets it he looks through the vacancy list once again to find a vacancy according to his new qualification.

5 Customer Behavior

Customer $h, h = 1,2,\ldots,H$ may be described by initial capital C_h and consuming profile F_{hi}. Customer may be also described by qualification and degree. Qualification L_i is defined by kind of production he is involved in. Assume that there are three levels of degree: degree1, degree2, degree3. Consumption profile has the following restrictions: consumed product cost must be less than or equal to $k_1 \cdot B_h$, where B_h is the income which includes salary, dividends and profits from stock market operations or redundancy relieves (for those who are unemployed).

The volume of the consumed products combination should not be lower than customer's survival level. Customer adjusts his level of consumption according to his profits. Different products/consumption level is defined by functions such as sine wave (seasonal fluctuation), large period sine wave (age fluctuation), almost constant function, exponential decreasing function, exponential increasing (popular) function, logical function.

Current wealth is distributed in the following way:

- Taxes make up γB_h, where γ - taxes rate, $0 \leq \gamma \leq 1$;
- Current expenses βB_h , where β - $0 \leq \beta \leq 1$;
- Savings, aimed for requalification or education. Amount, remaining after taxes paying $(1-\gamma)B_h$. For current expenses $\beta(1-\gamma)B_h$. $C_{edu} = 1-\beta(1-\gamma)B_h$ remains for education. Market basket cost of the customer h for period T is $\sigma_h = \sum_{t-1}^{T}\sum_{i-1}^{N}\phi_{it}F_{hit}$ and should not exceed βB_h, i.e. $\sigma_h \leq \beta B_h$. If this condition is not satisfied the customer adjust his consumption the following way: $F_{hit+1} = F_{hit} + \alpha(\sigma_h - \beta B_h)$, and checking survival condition $E_h = \sum_{t-1}^{T}\sum_{i-1}^{N} F_{hit} \cdot C_i \geq E^0$.

If this condition is not satisfied during period T_{max} the customer leaves the model. In the case when the customer is unemployed he can browse the vacancies list on the labor market. If he finds a vacancy of the enterprise j corresponding to his qualification in the list, he can make a contract with this enterprise, and after that he will be included into the salary list. If he buys enterprise's shares he will also be included into the list of those who obtain dividends. To improve his level of earning he may

increase his degree (if it is not maximum) using qualifier's service and paying the necessary sum S_{edu} under condition that $C_{edu} \geq S_{edu}$. If there is no vacancy for his qualification in the vacancies list he can get a new qualification using qualifier's services.

The aim of each customer is to increase its quality of life. The quality of life in the model consists of wealth and value of utility function corresponding to a customer's individual consumption profile. To increase his quality of life a customer uses his beliefs. According to them he can improve his quality of life by the following actions:

- constant job seeking according to a customer's qualification and degree (including requalification);
- buying/selling shares on the stock market and getting the dividends;
- saving money in a bank account;
- becoming an enterprise owner by buying its controlling interest

The customer strategy is the following. First of all the customer makes up a list of products that he can afford to buy at a definite sum not depending on whether he needs it or not. After that he sorts the price list in descending order by the value of utility function. Then using this sorted list the customer buys a product that gives the maximum increment of utility function and acts like that until he depletes the available financial resources.

6 Stock Market Functioning

Stock market simulation was previously described in the papers [14-15] which, in turn, was based on the model described by Li and Rosser [9]. New factors were introduced in the mentioned model. "Bad" or "good" news arise at each moment. There exists the memory in the model, which determines the rate at which news are forgotten. In such a way, the news background is being formed by addition of decaying news intensities. The news background modifies the fundamental value of current market price according to the logistic law. The insiders at moment $t-1$ know the prices at the moment t. The market simulation by means of the proposed model shows that, in case of "good" news, the stock-market prices are rising, and in the case of "bad" news, the prices are falling. Moreover, the parameter that determines the news-forgetting rate changes the picture of rising and falling prices. The model also shows, that the effect of insiders' activity depends on the return volume extracted by him, and when insiders' return approaches some crucial value, the fundamental value v abruptly falls down, and further with the increasing insiders' pressure the market explodes.

Stock market involves purchase and sale (trading) of securities emitted by enterprises. Each enterprise can emit only one kind of security. As different types of securities are traded independently on the stock market we can describe the trading process as if there was only one type of securities on the market. According to his

trading strategy a trader can belong to one of the three categories - fundamentalists, chartists (noise traders), and insiders (in special cases). Fundamentalist traders may change their strategy and pass into chartist category and vice versa in case the former strategy does not bring them enough profit.

To capture the significant contribution of news analysis to the decision making, we added the news background to our model, it replaced or added to the ”market noise system”. For each day, k random news events ξk are generated, whose value (or intensity) depends on the position of the ”good-bad” slider, and the news sign (”bad” or ”good”) depends on the slider displacement from the neutral position. The news in this model have the property of being accumulated in time, but the strength of news is decaying with time. The accumulated news comprise the news background, which may be neutral, positive, or negative, depending on the sign of accumulated news.

The term ”insiders” reflects the fact that the specific information they possessed until the current moment was for internal use only inside a limited number of users. As a result, these persons, getting the insiders' information before its official publication, have an opportunity to make the certain moves at the stock market for obtaining superior profits.

7 Enterprise Investment Strategy

It is considered that the agent ENTERPRISE acts in perfect competition.

All the enterprises in the model produce the same types of goods. Some of them belong to the group of basic consumer goods (food, clothes), others are articles of secondary necessity (mobile phones) and others are luxury goods (automobiles, cottages) at each time horizon. In the model we use the Cobb-Douglas production function. An enterprise is trying to find an optimal volume of output taking into account the balance between total income and total costs. An enterprise accomplishes its investment strategies in an effort to remove the mismatch between demand and supply on the market.

The customer driven enterprise strategy in the model is realized as follows. Three activity partitions display three types of enterprise strategies. The first one corresponds to the case when all profit obtained by an enterprise is divided among the employees according to their qualification and degree. The second one shows the case when an enterprise gets a positive profit and uses it to increase the production level according to the customers' demand. The third one describes the situation when an enterprise has a negative profit and it covers its losses by partly selling its capacity. If nevertheless the losses are not covered and the enterprise's capital becomes negative the enterprise is declared bankrupt and leaves the model.

8 Results

The model is implemented and simulated within the AnyLogic environment[20].

To present the results of the research on the modification of the wealth distribution function we examined condition of wealth distribution during the first twelve years of model run with the following initial parameters: 300 customers and 5 enterprises in the model. The data on the customers' wealth were imported and processed in Statistica 8.0 (http://statsoft.com/). The following figures show how the wealth distribution changes from the initial period to the twelfth year (624 model time steps).

After 52 time steps that makes one year the wealth distribution modifies from rectangular to a normal one as you can see in Figures 5 and 6:

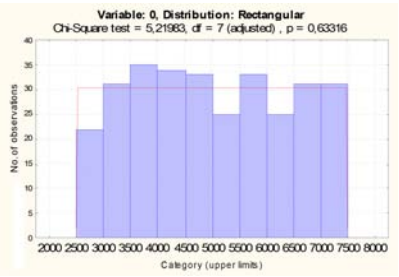

Fig. 5 Original wealth distribution.

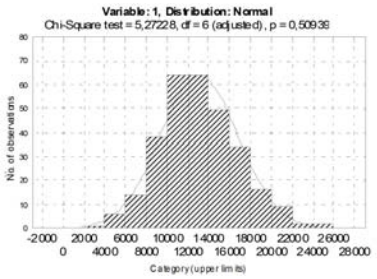

Fig. 6 Wealth distribution one year later.

During another nine model years we'll follow up how that picture changes. The very middle of the time period analyzed in the model, six years, is characterized by a gamma wealth distribution. It is evident from Figure 8 that wealth is mainly accumulated by the middle class customers whilst the number of people belonging to the poor class declines and the wealthy class, to the contrary, increases. This trend is preserved for the several years to come.

As you can see in the Figure 9 (that shows the condition of the model in the eight year) the wealth distribution switches back to normal as it was recorded seven years before that. Again, the middle class as well as the wealthy class are growing unlike the poor one.

What we can observe in the last chart (Figure 10) is that the wealth distribution returns to the uniform distribution after twelve years of running the model.

So we can observe the following steps in the evolution of the wealth distribution: uniform - normal - gamma - normal - uniform. As the production development and assortment innovations are not introduced in the model the system becomes stabilized as well as the wealth distribution.

We also found out that a group of customers with a much higher income (5-6 times higher than an average income) and a gap dividing the two groups of customers appear once we set some critical tax level (24%). This result is proved out

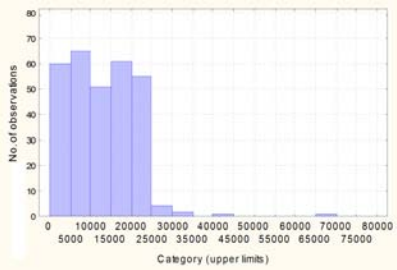

Fig. 7 Wealth distribution three years later.

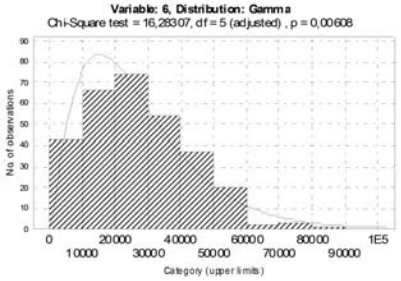

Fig. 8 Wealth distribution six years later.

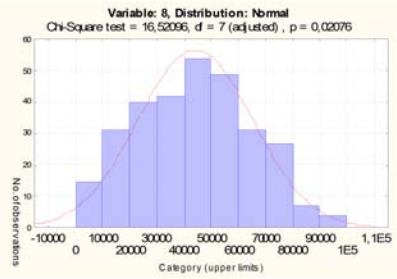

Fig. 9 Wealth distribution eight years later.

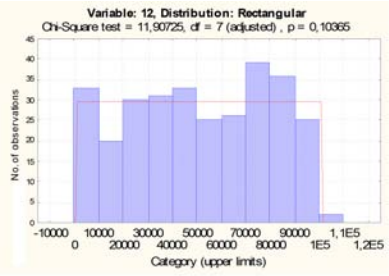

Fig. 10 Wealth distribution twelve years later.

both if we set 300 customers (5 enterprises), 600 customers (10 enterprises), and 1000 customers (10 enterprises) initially. The effect can be better observed with the initial parameters 600 customers and 10 enterprises after twelve years of model years.

By the way the influence of tax level on the type of distribution function is mentioned in the work of Chakrabarty. In a trading model with taxation and redistribution where τ is fraction of money taxed from traders it was shown [14] that exponential distribution transfers to gamma function as τ goes up and then after a threshold, it returns to exponential for higher values of τ.

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