A HYBRID SIMULATION OPTIMIZATION APPROACH FOR SUPPLY CHAINS

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Abstract

The main idea of our approach is to combine discrete-event simulation and exact optimization for supply chain network models. Simulation models are constructed in order to mimic a real system including all necessary stochastic and nonlinear elements. Such simulation models are used as proving grounds for analyzing and improving a real situation on a trial-and-error basis. A traditional optimization method on top of a simulation model has major disadvantages: The optimization method uses the simulation model as a black-box. Information about the structure of the problem is not available and cannot be used for an intelligent optimization strategy.
On the other hand pure optimization models used for planning scenarios are usually built on a very abstract level neglecting possibly important nonlinear and stochastic properties. This is necessary, because otherwise the resulting complex optimization models cannot be solved and are therefore of no use.
We present a possible way out of this dilemma by combining the use of a simple optimization model within the framework of a complex simulation model. The embedded optimization model is used to improve the overall performance by adapting decision rules. Based on the idea of a fixed-point iteration we couple a discrete-event model and its linearized deterministic representation and solve it alternately. Already after a few iterations we can gain convergence to good quality solutions within much less computational time than traditional optimization approaches.

Keywords: discrete-event simulation, optimization, supply chains, improvement strategies, decision support systems

Presenting Author's biography

Christian Almeder works currently as an assistant professor at the Institute for Business Administration at the University of Vienna. He made his Ph.D. in Applied Mathematics at Institute for Analysis and Technical Mathematics and worked several years as a research assistant at the Department of Operational Research at the Vienna University of Technology. His publications stretches from mathematical simulation in the fields of biomedical engineering and epidemiology to operations research in production and logistics.

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1 Introduction

Supply chain networks nowadays are used to be fairly complex, such that an intensive planning is complicated but necessary. Classical optimization models in most cases are not appropriate, because of the inability of representing stochastic or highly complex relations between the different entities. Although recent developments in research try to consider more features the computational effort for approximating optimal solutions is usually prohibitive. Discrete-event simulation offers a broad variety of tools to incorporate complex and stochastic behavior, but improving strategies for certain objectives are mainly restricted to a trail-and-error procedure [4,5]. Optimization methods using simulation as a black-box are commonly used, but they lack of structural information about the problem [3]. Furthermore long computational times for evaluating the (simulation-based) objective makes classical search procedures inefficient.

In this paper we develop a new approach for using a linear program formulation in the context of a discrete-event simulation. Our investigations are based on a general supply chain network model with different facilities (suppliers, manufacturers, distributors) and different transportation modes connecting those facilities. The aim is to reduce costs by simultaneously optimizing the production/transportation schedule and reducing inventory levels.

There are only a few papers dealing with optimization of network flows in the context of supply chain management simulation. Yaged [13] discusses in his paper a static network model which includes nonlinearities. He tries to optimize the flow by solving a linearized version of the network and improve the flow in the network. Paraschis [8] discusses several different possibilities to linearize such networks and Fleischmann [2] presents several applications of network flow models, which are solved through linearization. But all three papers do not include any stochastic elements. Lee and Kim [6] show a real combination of simulation and optimization for the case of a production-distribution system. They use simulation to check the result of the simpler optimization model in a more realistic environment and to update the parameters for the optimization. After several iterations they end up with a solution of the optimization model which is also within the constraints of the stochastic simulation model. Truong and Azadivar [12] developed an environment for solving supply chain design problems, where they combine simulation with genetic algorithms and mixed-integer programs. But they remain on a strategic level including only a few decision to be optimized about facility location and partner selection. Stäblein et al. [11] developed a simulation tool for supply chain networks. They included optimization in the sense of a advanced planning system for each actor in the supply chain.

In some previous work [1,7,9,10] we have described details about the optimization model of the supply chain. In this paper we will concentrate on the simulation model and its connection to the optimization model.

2 Problem description

The supply chain considered for investigation consists of three different actors: suppliers, producers, and customers. It is assumed that a central planning for all actors except the customers is possible. So we discuss the situation of a company-internal supply chain or a supply chain with a single leading partner. The actors are connected through transportation connections, so the network structure is predefined. Suppliers act as a source for raw materials. New raw materials are generated given to certain predefined rate. The raw materials are stored until an order from another actor is received and the materials are delivered. Producers can order raw materials at suppliers or other producers and store it in an input storage. According to a given bill-of-materials, the producers transform raw materials into new products and store them in an output storage waiting to be delivered to customers or other producers. The customers have a certain predefined demand rate. According to this demand the customers order products at producers. Early delivers can be stored at some cost, late deliveries are penalized.

The total cost of operation is used as the objective function to be minimized. These costs include storage costs, transportation costs, production costs and penalty costs for late deliveries to the customers.

3 Methodology

The supply chain is represented as a discrete-event simulation model. A simplified version is captured using an optimization model, which might be a linear program, a mixed integer linear program, or any kind of optimization model which is small enough to gain solutions with a reasonable computational effort. At first we perform several simulation runs in order to be able to estimate the parameters for the optimization model. This might be done by simply calculating the mean, or by making a distinct statistical analysis and determine appropriate values based on the sample. This second procedure is necessary for critical parameters which have a strong influence on the objective. After this step we can perform a solution algorithm to obtain a result of the optimization model. This result is transformed into decision rules for the simulation model in order to improve the overall performance of the system. Now we start again with further simulation experiments and analyze if the picture has changed in terms of the objective as well as in the parameters we estimated in the previous iteration. Due to the changed decision rules we might be in a completely different situation which

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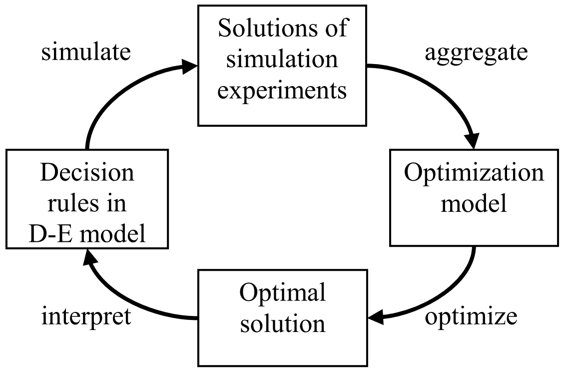

Fig. 1 Interaction between simulation and optimization.

necessitates a recalculation of the parameters for the optimization model. So we are looping between the simulation and the optimization model until we converge to a stable solution (see Figure 1). The question of convergence of this procedure cannot be answered in general, since it depends on the structure of the parameter aggregation process and the generation algorithm for new decision rules. Although we are able to construct special examples, where no convergence is possible, all computations on realistic and real-world test cases lead to very fast convergence (see Section 5).

4 Implementation

We have chosen AnyLogic by XJ Technologies for developing a discrete-event simulation model and XpressMP by Dash Optimizations for the optimization model. Both models are connected via a Microsoft Access database.

4.1 Simulation model

The simulation model is constructed as a library including several different modules representing the different parts of the supply chain network:

- suppliers providing raw materials;
- customers who demand certain products at a specific time;
- production sites where production, stocking, and transshipment takes place.
- transportation connections between actors of the supply chain.

Furthermore, we need a special control module necessary for controlling the simulation experiments as well as organizing the communication with the optimization model implemented in XpressMP

4.1.1 Module *Supplier*

This module represents a supplier in the supply chain. It is used to generate certain raw materials, store them, and deliver them if demanded. It has one input port to receive orders for products and one output port to send out products. If this module receives an order through the input port, it sends the requested amount of products via the output port. If the amount exceeds the current inventory level, only the available amount is sent. As soon as new products arrive in the inventory they are delivered until the whole order has been fulfilled. The costs arising in this module are only inventory costs for storing products prior to delivery. These costs may have any user-defined functional form. The raw materials generated in this module per period are assumed to be given.

4.1.2 Module *Production*

This module is the core of the whole model. It represents a production site and consists of an input and an output storage. Items are either transformed into new items or simply transferred to the output storage. This module has one input port and one output port for orders, as well as one input and one output port for products. The input storage is replenished by ordering products via the output port for orders from a supplier or another production module. The ordering policy may be either autonomous (e.g. an (s,S)-policy or any user-defined policy) or it is determined by the result of the linear model. Products are received through the product input port and stored in the input inventory. The production of new products is initiated by an order placed by the output inventory. The delay for production is a user-defined function. It may contain stochastic elements and depend on other parameters (e.g. the current load). Production has limited capacities and furthermore production is restricted to the availability of raw materials. If these capacities do not allow producing a lot as a whole, it is split into several batches. Through the input order port the module receives orders from other production or customer modules. Products are sent through the output product port according to these orders and based on availability. Costs arise in this model for inventory holding (input and output) and for production.

4.1.3 Module *Customer*

According to a given demand table, the customer orders the products at the production sites. The customer has an input inventory, from which the demand is satisfied. The inventory level can be negative (shortages) as well as positive (oversupply). It has one output port for sending orders and one input port for receiving products. The orders are sent either according to the demand table (including a standard delay time for transportation) or according to the solution of the linear model. At this module shortage costs as well as penalty cost for positive inventory occur.

4.1.4 Module *Transport*

This module is used to transport products between different modules. It receives products through its single input port and sends it (according to some time delay) through the output port to the next module (Production or Customer). It has a limited capacity and organizes the transports according to a FIFO rule. It is also possible to split shipments if the available capacity does not allow a single shipment. The user-defined time delay may be stochastic and may depend on other parameters. User-defined costs arise for finished transportation and may include transportation time, amounts, and fixed charge parts.

4.2 Optimization model

The optimization model is developed as a linear program. The decision variables are production schedules and delivery plans for each node of the supply chain network. The parameters used for the optimization model can be classified into 3 classes according to their source:

- General network parameters (e.g. capacity limitations): This parameters are describing properties of the supply chain network, which are used in the optimization model as well as in the simulation model
- Non-critical simulation parameters (e.g. cost factors): These are parameters which are calculated based on simulation experiments, but they are not critical for the operation of the supply chain.
- Critical simulation parameters (e.g. transportation delays): These parameters are based on simulation experiments and are critical for the operation of the supply chain.

A detailed description of the linear model can be found in [16].

4.3 Connecting simulation and optimization

The simulation model is used as the master process. It controls the communication between the simulation and the optimization model. Figure 2 shows the logical connection between the simulation and the optimization model and the database in the middle.

To initiate the optimization process in our system, the simulation model loads all necessary data about the network structure and its capacities from the database. Then several simulation runs are performed using some autonomous decision rules (like an (s,S)-policy for the replenishment) instead of the missing rules given by the solution of the optimization model in later iterations. The results of these runs (transportation delays, per unit costs, …) are stored and after the last run, the mean costs and newly computed delays (based on mean and on variance) are stored in the database. Afterwards XPressMP ist executed. It loads the general data and the simulation results from the database, computes the solution of the optimization model and stores the results (ordering and delivery plans, production and transfer schedules, …) in the database. Then the simulation model starts again several experiments using now the newly computed decision rules based on the solution of the optimization model. The whole iterative solution procedure is described in Table 1.

5 Results

The theory on fixed-point methods cannot guarantee convergence for our solution method and furthermore, we are able to construct a special counterexample where we generate a cyclic behaviour. Nevertheless, we can observe convergence in all our empirical tests, i.e. the gap between the results of the optimization model and the simulation model is decreasing and after 3-4 iterations it reaches already an acceptable low level of less than 1%.

Using different test instances we are able to demonstrate that this method leads to high quality results within a short computational time compared with traditional methods. In Table 2 we present a comparison of our hybrid optimization with a pure optimization approach. The test instances consist of a simple supply chain with one supplier, one producer, and one customer. The test instances are classified with respect to the customer demand (high and low demand). The final solutions were evaluated by averaging the results of 20 simulation runs. The simulation model contains many stochastic and nonlinear features which cannot be captured in the linear program used as the optimization model within our simulation. For the results of the complex optimization approach in the third column we developed an according mixed-integer program, which captures all nonlinear elements of the supply chain. Furthermore, the stochastic elements where substituted by estimations based on the known mean and variance.

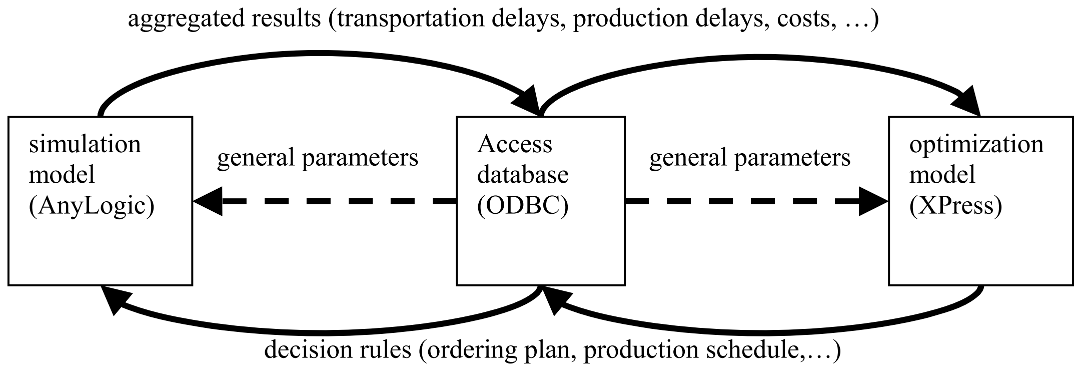

Fig. 2: Pseudo code for the combined simulation optimization approach.

Tab. 1: Pseudo code for the hybrid simulation optimization approach.

```
Load necessary simulation parameters from the database
Perform several simulation runs using autonomous decision rules
Aggregate results and store them in the database
while stopping criteria is not met
      Load aggregated parameters into LP/MIP-Solver
      Solve the optimization model
      Write new decision rules to the database
      Load new decision rules into simulation model
      Perform several simulation runs using these decision rules
      Aggregate results and store them in the database
end-while
```

Tab. 2: Comparison of total costs between our hybrid simulation optimization approach and complex optimization model for small stochastic test cases classified by the occurrence of customer demand (H – high demand, L – low demand)

Test	**sim-opt (<1 min.)**	**complex opt. (10-30 min)**	**Diff.**
S1-H	31324	30150	3.89%
S2-H	123632	112809	9.59%
S3-H	81224	65014	24.93%
S4-H	412138	414917	-0.67%
S5-H	280270	274762	2.00%
S6-H	1132866	1127708	0.46%
S7-H	62742	74730	-16.04%
S8-L	13835	13356	3.59%
S9-L	11240	10291	9.22%
S10-L	14354	12594	13.97%
S11-L	12919	11068	16.72%
S12-L	26437	20422	29.45%
Avg.-H	**303457**	**300013**	**3.45%**
Avg.-L	**15757**	**13546**	**14.59%**
Average	**183582**	**180652**	**8.09%**

The table indicates that our method is extremely fast compared with a traditional optimization approach. Although on average the simulation-based approach is a little bit worse regarding the solution quality, the results show a high variation of possible outcomes. So we may conclude that even a complex optimization may not lead to good results in some stochastic cases. Also the high penalty costs for late deliveries may lead to big differences of the total costs if only one or two orders are delayed.

The difference in the solution quality between high and low demand instances is due to the fact that with a mixed-integer program it is possible to gain positive effects from combining several production or transportation lots. This combination is not possible in the pure linear program used in our hybrid approach.

Furthermore, we tested our approach also with larger instances including 3 suppliers, 4 producers, 3 customers, and 16 transportation modules (see Figure 3). The size of this model prohibits the calculation of an optimal solution including all features. In spite, our hybrid approach, where we use only a simplified linear program within the simulation framework, can be easily applied with calculation times of only a few minutes.

6 Conclusions

In this paper we have presented a new approach that combines the advantages of complex simulation models and abstract optimization models. We have shown that our method is able to generate competitive solutions quickly, even compared with traditional planning approaches that are much more time consuming.

For situations with tight capacity constraints, we have seen that the use of a pure linear program without binary decisions in the optimization step of our simulation-based approach leads to high quality solutions within a very short time. Nevertheless, in situations with loose capacity constraints and highly nonlinear cost relations it is advisable to include some of these aspects also in the optimization model.

The question, which aspects should be included in the optimization model, is not completely answered yet. If more complex models are used, other fast solution methods (e.g. heuristics, metaheuristics …) should be taken into consideration.

Our simulation-based optimization method can be seen as a general framework, which might be applied to other stochastic planning problems.

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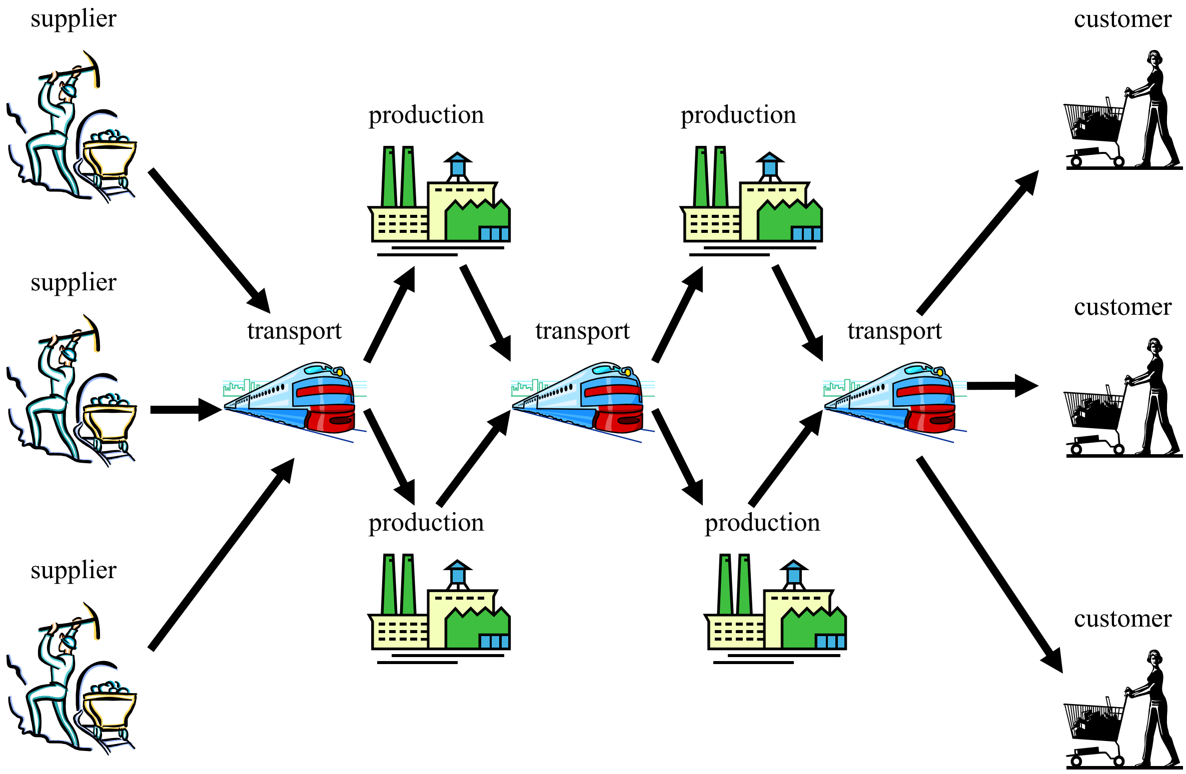

Fig. 3: Exemplary supply chain network.

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